Enhanced $K_L \to \pi^0 \nu\bar{\nu}$ from Direct CP Violation in $B \to K\pi$ with Four Generations

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Recent CP violation results in B decays suggest that Z penguins may have large weak phase. This can be realized by the four generation (standard) model. Concurrently, $B \to X_s \ell^+\ell^-$ and B_s mixing allow for sizable $V_{t's}^* V_{t'b}$ only if it is nearly imaginary. Such large effects in $b \leftrightarrow s$ transitions would affect $s \leftrightarrow d$ transitions, as kaon constraints would demand $V_{t'd} \neq 0$. Using $\Gamma(Z \to b\bar{b})$ to bound $|V_{t'b}|$, we infer sizable $|V_{t's}| \lesssim |V_{t'b}| \lesssim |V_{us}|$. Imposing ε_K, $K^+ \to \pi^+\nu\bar{\nu}$ and ε'/ε constraints, we find $V_{t'd}^* V_{t's} \sim$ few $\times 10^{-4}$ with large phase, enhancing $K_L \to \pi^0\nu\bar{\nu}$ to 5×10^{-10} or even higher. Interestingly, Δm_{B_d} and $\sin 2\Phi_{B_d}$ are not much affected, as $|V_{t'd}^* V_{t'b}| \ll |V_{td}^* V_{tb}| \sim 0.01$.

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Just 3 years after CP violation (CPV) in the B system was established, direct CP violation (DCPV) was also observed in $B^0 \to K^+\pi^-$ decay, $\mathcal{A}_{K^+\pi^-} \sim -0.12$. A puzzle emerged, however, that the charged $B^+ \to K^+\pi^0$ mode gave no indication of DCPV, and is in fact a little positive, $\mathcal{A}_{K^+\pi^0} \gtrsim 0$. Currently, $\mathcal{A}_{K^+\pi^0} - \mathcal{A}_{K^+\pi^-} \simeq 0.16$, and differs from zero with 3.8σ significance [1].

The amplitude $\mathcal{M}_{K^+\pi^-} \simeq P + T$ is dominated by the strong penguin (P) and tree (T) contributions, while the main difference $\sqrt{2}\mathcal{M}_{K^+\pi^0} - \mathcal{M}_{K^+\pi^-} \simeq P_{\rm EW} + C$ is from electroweak penguin (EWP, or $P_{\rm EW}$) and color-suppressed tree (C) contributions which are subdominant. Thus, $\mathcal{A}_{K^+\pi^0} \sim \mathcal{A}_{K^+\pi^-}$ was anticipated by all models. As data indicated otherwise, it has been stressed [2] that the C term could be much larger than previously thought, effectively cancelling against the CPV phase in T, leading to $\mathcal{A}_{K\pi^0} \to 0$. While this may well be realized, a very large C (especially if $\mathcal{A}_{K\pi^0} > 0$) would be a surprise in itself.

In a previous paper [3], we explored the possibility of New Physics (NP) effects in $P_{\rm EW}$, in particular in the 4 generation standard model (SM4, with SM3 for 3 generations). A sequential t' quark could affect $P_{\rm EW}$ most naturally for two reasons. On one hand, the associated Cabibbo-Kobayashi-Maskawa (CKM) matrix element product $V_{t's}^* V_{t'b}$ could be large and imaginary; on the other hand, it is well known that $P_{\rm EW}$ is sensitive to $m_{t'}^2$ in amplitude, and heavy t' does not decouple.

Using the PQCD factorization approach at leading order [4], which successfully predicted $\mathcal{A}_{K^+\pi^-} < -0.1$ (and C was not inordinately large), we showed that $\mathcal{A}_{K^+\pi^0} \gtrsim 0$ called for sizable $m_{t'} \gtrsim 300$ GeV and large, nearly imaginary $V_{t's}^* V_{t'b}$. As the $m_{t'}$ dependence is similar, we also showed that data on $B \to X_s \ell^+\ell^-$ and B_s mixing concurred, in the sense that large t' effect is allowed *only if* $V_{t's}^* V_{t'b}$ is nearly imaginary. Applying the latter two constraints, however, $m_{t'}$ and $V_{t's}^* V_{t'b}$ become highly constrained. In the following, we will take [3]

$$m_{t'} \cong 300 \text{ GeV}, \; V_{t's}^* V_{t'b} \equiv r_{sb}\, e^{i\phi_{sb}} \simeq 0.025\, e^{i\,70^\circ}, \quad (1)$$

as exemplary values for realizing $\mathcal{A}_{K^+\pi^0} - \mathcal{A}_{K^+\pi^-} \gtrsim 0.10$, without recourse to a large C contribution.

Comparing with $|V_{cs}V_{cb}| \simeq 0.04$, $r_{sb} \sim 0.025$ is quite sizable. In our $b \to s$ study, we had assumed [3] $V_{t'd} \to 0$ out of convenience, so as to decouple from $b \to d$ and $s \to d$ concerns. The main purpose of this note, however, is to show that, in view of the large r_{sb} and ϕ_{sb} values given in Eq. (1), $V_{t'd} = 0$ is untenable, and one must explore $s \to d$ and $b \to d$ implications. The reasoning is as follows. Since a rather large impact on $V_{ts}^* V_{tb}$ is implied by Eq. (1), if one sets $V_{t'd} = 0$, then $V_{td}^* V_{ts}$ would still be rather different from SM3 case. With our current knowledge of m_t, the ε_K parameter would deviate from the well measured experimental value. Thus, a finite $V_{t'd}$ is needed to tune for ε_K.

We find that the kaon constraints that are sensitive to t' (i.e. $P_{\rm EW}$-like), viz. $K^+ \to \pi^+\nu\bar{\nu}$, $K_L \to \mu^+\mu^-$, ε_K, and ε'/ε can all be satisfied. Interestingly, once kaon constraints are satisfied, we find little impact is implied for $b \leftrightarrow d$ transitions, such as Δm_{B_d} and $\sin 2\Phi_{B_d}$. That is, $V_{t'd} \to 0$ works approximately for $b \to d$ transitions, for current level of experimental sensitivity. The main outcome for $s \to d$ and $b \to d$ transitions is the enhancement of $K_L \to \pi^0\nu\bar{\nu}$ mode by an order of magnitude or more, to beyond 5×10^{-10}.

With four generations, adding $V_{t's}^* V_{t'b}$ extends the familiar unitarity triangle relation into a quadrangle,

$$V_{us}^* V_{ub} + V_{cs}^* V_{cb} + V_{ts}^* V_{tb} + V_{t's}^* V_{t'b} = 0. \quad (2)$$

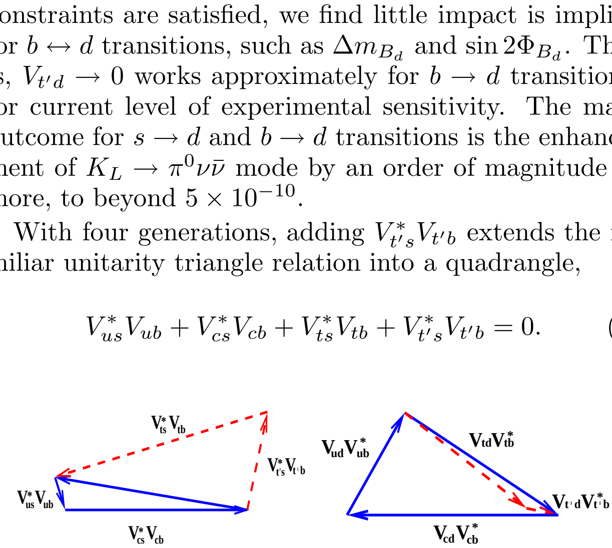

FIG. 1: Unitarity quadrangles of (a) Eq. (2), with $|V_{us}^* V_{ub}|$ exaggerated; (b) Eq. (17), where actual scale is $\sim 1/4$ of (a). Adding $V_{t's}^* V_{t'b}$ (dashed) according to Eq. (1) drastically changes the invariant phase and $V_{ts}^* V_{tb}$ from the SM3 triangle (solid), but from Eq. (16), the dashed lines for $V_{td} V_{tb}^*$ and $V_{t'd} V_{t'b}^*$ can hardly be distinguished from SM3 case.

Using SM3 values for $V_{us}^*V_{ub}$, $V_{cs}^*V_{cb}$ (validated later by our $b \to d$ study), since they are probed in multiple ways already, and taking $V_{t's}^*V_{t'b}$ as given in Eq. (1), we depict Eq. (2) in Fig. 1(a). The solid, rather squashed triangle is the usual $V_{us}^*V_{ub} + V_{cs}^*V_{cb} + V_{ts}^*V_{tb} = 0$ in SM3. Given the size and phase of $V_{t's}^*V_{t'b}$, one sees that the invariant phase represented by the area of the quadrangle is rather large, and $V_{ts}^*V_{tb}$ picks up a large imaginary part, which is very different from SM3 case. Such large effect in $b \to s$ would likely spill over into $s \to d$ transitions, since taking V_{tb} as real and of order 1, one immediately finds the strength and complexity of $V_{td}^*V_{ts}$ would be rather different from SM3, and one would need $V_{t'd}^*V_{t's} \neq 0$ to compensate for the well measured value for ε_K.

Note from Fig. 1(a) that the usual approximation of dropping $V_{us}^*V_{ub}$ in the loop remains a good one. To face $s \to d$ and $b \to d$ transitions, however, one should respect unitarity of the 4×4 CKM matrix $V_{\rm CKM}$. We adopt the parametrization in Ref. [5] where the third column and fourth row is kept simple. This is suitable for B physics, as well as for loop effects in kaon sector. With V_{cb}, V_{tb} and $V_{t'b}$ defined as real, one keeps the SM3 phase convention for V_{ub}, now defined as

$$\arg V_{ub}^* = \phi_{ub}, \tag{3}$$

which is usually called ϕ_3 or γ in SM3. We take $\phi_{ub} = 60^\circ$ as our nominal value [6]. This can in principle be measured through tree level processes such as the $B \to DK$ Dalitz method [7]. The two additional phases are associated with $V_{t's}$ and $V_{t'd}$, and for the rotation angles we follow the PDG notation [8]. To wit, we have

$$\begin{aligned} V_{t'd} &= -c_{24}c_{34}s_{14}e^{-i\phi_{db}}, &(4)\\ V_{t's} &= -c_{34}s_{24}e^{-i\phi_{sb}}, &(5)\\ V_{t'b} &= -s_{34}, &(6)\end{aligned}$$

while $V_{t'b'} = c_{14}c_{24}c_{34}$, $V_{tb} = c_{13}c_{23}c_{34}$, $V_{cb} = c_{13}c_{34}s_{23}$ are all real. With this convention for rotation angles, from Eq. (3) we have $V_{ub} = c_{34}s_{13}e^{-i\phi_{ub}}$.

Analogous to Eq. (1), we also make the heuristic but redundant definition of

$$V_{t'd}^*V_{t'b} \equiv r_{db}\, e^{i\,\phi_{db}},\ V_{t'd}^*V_{t's} \equiv r_{ds}\, e^{i\,\phi_{ds}}, \tag{7}$$

as these combinations enter $b \to d$ and $s \to d$ transitions. Inspection of Eqs. (1), (4 –6) gives the relations

$$r_{db}r_{sb} = r_{ds}s_{34}^2,\ \phi_{ds} = \phi_{db} - \phi_{sb}. \tag{8}$$

As we shall see, $s \to d$ transitions are much more stringent than $b \to d$ transitions, hence we shall turn to constraining r_{ds} and ϕ_{ds}.

Before turning to the kaon sector, we need to infer what value to use for $s_{34} = |V_{t'b}|$, as this can still affect the relevant physics through unitarity. Fortunately, we have some constraint on s_{34} from $Z \to b\bar{b}$ width, which receives special t (and hence t') contribution compared to other $Z \to q\bar{q}$, and is now suitably well measured.

Following Ref. [9] and using $m_{t'} = 300$ GeV, we find

$$|V_{tb}|^2 + 3.4|V_{t'b}|^2 < 1.14. \tag{9}$$

Since all c_{ij}s except perhaps c_{34} would still likely be close to 1, we infer that $s_{34} \lesssim 0.25$. We take the liberty to nearly saturate this bound ($\Gamma(Z \to b\bar{b})$ is close to 1σ above SM3 expectation), by *imposing*

$$s_{34} \simeq 0.22, \tag{10}$$

to be close to the Cabibbo angle, $\lambda \equiv |V_{us}| \cong 0.22$. Note that Eq. (10) is somewhat below the expectation of "maximal mixing" of $s_{34}^2 \sim 1/2$ between third and fourth generations. Combining it with Eq. (1), one gets $|V_{t's}| \sim 0.11 \sim \lambda/2$. Its strength would grow if a lower value of $s_{34} \lesssim \lambda$ is chosen, which would make even greater impact on $s \to d$ transitions.

Using current values [8] of V_{cb} and V_{ub} as input and respecting full unitarity, we now turn to the kaon constraints of $K^+ \to \pi^+\nu\bar{\nu}$, ε_K, $K_L \to \mu^+\mu^-$, and ε'/ε. The first two are short-distance (SD) dominated, while the last two suffer from long-distance (LD) effects.

Let us start with $K^+ \to \pi^+\nu\bar{\nu}$. The first observed event [10] by E787 suggested a sizable rate hence hinted at NP. The fourth generation would be a good candidate, since the process is dominated by the Z penguin. Continued running, including E949 data (unfortunately not greatly improving accumulated luminosity), has yielded overall 3 events, and the rate is now $\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu}) = (1.47^{+1.30}_{-0.89}) \times 10^{-10}$ [11]. This is still somewhat higher than the SM3 expectation of order 0.8×10^{-10}.

Defining $\lambda_q^{ds} \equiv V_{qd}V_{qs}^*$ and using the formula [12]

$$\begin{aligned}\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu}) \ =\ \kappa_+ \Bigg| &\frac{\lambda_c^{ds}}{|V_{us}|}P_c + \frac{\lambda_t^{ds}}{|V_{us}|^5}\eta_t X_0(x_t) \\ &+ \frac{\lambda_{t'}^{ds}}{|V_{us}|^5}\eta_{t'} X_0(x_{t'})\Bigg|^2, \end{aligned}\tag{11}$$

we plot in Fig. 2 the allowed range (valley shaped shaded region) of r_{ds}–ϕ_{ds} for the 90% confidence level (C.L.) bound of $\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu}) < 3.6 \times 10^{-10}$. We have used [12] $\kappa_+ = (4.84 \pm 0.06) \times 10^{-11} \times (0.224/|V_{us}|)^8$ and $P_c = (0.39 \pm 0.07) \times (0.224/|V_{us}|)^4$. We take the QCD correction factors $\eta_{t^{(\prime)}} \sim 1$, and $X_0(x_{t^{(\prime)}})$ evaluated for $m_t = 166$ GeV and $m_{t'} = 300$ GeV. We see that r_{ds} up to 7×10^{-4} is possible, which is not smaller than the SM3 value of 4×10^{-4} for $|V_{td}^*V_{ts}|$.

The SD contribution to $K_L \to \mu^+\mu^-$ is also of interest. The $K_L \to \mu^+\mu^-$ rate is saturated by the absorptive $K_L \to \gamma\gamma \to \mu^+\mu^-$, while the off-shell photon contribution makes the SD contribution hard to constrain. To be conservative, we use the experimental bound of $\mathcal{B}(K_L \to \mu^+\mu^-)_{\rm SD} < 3.7 \times 10^{-9}$ [13]. It is then in general less stringent than $K^+ \to \pi^+\nu\bar{\nu}$, although the generic constraint on r_{ds} drops slightly. We do not plot this constraint in Fig. 2.

The rather precisely measured CPV parameter $\varepsilon_K = (2.284 \pm 0.014) \times 10^{-3}$ [8] is predominantly SD. It maps

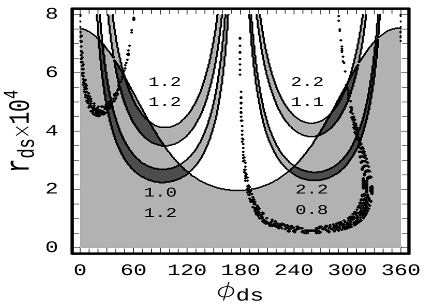

FIG. 2: Allowed region from $K^+ \to \pi^+\nu\bar{\nu}$ (valley shaped shaded region), ε_K (simulated dots) and ε'/ε (elliptic rings) in r_{ds} and ϕ_{ds} plane, as described in text, where $V^*_{t'd}V_{t's} \equiv r_{ds}\,e^{i\phi_{ds}}$. For ε'/ε, the rings on upper right correspond to $R_6 = 2.2$, and $R_8 = 0.8$, 1.1 (bottom to top), and on upper left, $R_6 = 1.0$, 1.2 (bottom to top), $R_8 = 1.2$.

out rather thin slices of allowed regions on the r_{ds}–ϕ_{ds} plane, as illustrated by dots in Fig. 2, where we use the formula of Ref. [9] and follow the treatment. Note that r_{ds} up to 7×10^{-4} is still possible, for several range of values for ϕ_{ds}. This is the aforementioned effect that extra CPV effects due to large ϕ_{sb} and r_{sb} now have to be tuned by t' effect to reach the correct ε_K value. We have checked that Δm_K makes no additional new constraint.

The DCPV parameter, $\mathrm{Re}\,(\varepsilon'/\varepsilon)$, was first measured in 1999 [14], with current value at $(1.67 \pm 0.26) \times 10^{-3}$ [8]. It depends on a myriad of hadronic parameters, such as m_s, $\Omega_{\rm IB}$ (isospin breaking), and especially the non-perturbative parameters R_6 and R_8, which are related to the hadronic matrix elements of the dominant strong and electroweak penguin operators. With associated large uncertainties, we expect ε'/ε to be rather accommodating, but for specific values of R_6 and R_8, some range for r_{ds} and ϕ_{ds} is determined.

We use the formula

$$\mathrm{Re}\,\frac{\varepsilon'}{\varepsilon} = \mathrm{Im}\,(\lambda_c^{ds})P_0 + \mathrm{Im}\,(\lambda_t^{ds})F(x_t) + \mathrm{Im}\,(\lambda_{t'}^{ds})F(x_{t'}), \tag{12}$$

where $F(x)$ is given by

$$F(x) = P_X X_0(x) + P_Y Y_0(x) + P_Z Z_0(x) + P_E E_0(x). \tag{13}$$

The SD functions X_0, Y_0, Z_0 and E_0 can be found, for example, in Ref. [15], and the coefficients P_i are given in terms of R_6 and R_8 as

$$P_i = r_i^{(0)} + r_i^{(6)} R_6 + r_i^{(8)} R_8, \tag{14}$$

which depends on LD physics. We differ from Ref. [15] by placing P_0, multiplied by $\mathrm{Im}\,(\lambda_c^{ds})$, explicitly in Eq. (12). In SM4, one no longer has the relation $\mathrm{Im}\lambda_c^{ds} = -\mathrm{Im}\lambda_t^{ds}$ that makes $\mathrm{Re}\,(\varepsilon'/\varepsilon)$ proportional to $\mathrm{Im}(\lambda_t^{ds})$. We take the $r_i^{(j)}$ values from Ref. [15] for $\Lambda^{(4)}_{\overline{MS}} = 310$ MeV, but reverse the sign of $r_0^{(j)}$ for above mentioned reason. Note that $\mathrm{Re}\,(\varepsilon'/\varepsilon)$ depends linearly on R_6 and R_8. For fixed SD parameters $m_{t'}$ and $\lambda_{t'}^{ds} = V_{t'd}V^*_{t's}$, one may adjust for solutions to $K^+ \to \pi^+\nu\bar{\nu}$ and ε_K.

For the "standard" [15] parameter range of $R_6 = 1.23 \pm 0.16$ and $R_8 = 1.0 \pm 0.2$, we find $R_8 \sim 1.2$ and $R_6 \sim$ 1.0–1.2 allows for solutions at $r_{ds} \sim (5–6) \times 10^{-4}$ with $\phi_{ds} \sim +(35^\circ–50^\circ)$, as illustrated by the elliptic rings on upper left part of Fig. 2. For $R_6 = 2.2 \pm 0.4$ found [16] in $1/N_C$ expansion at next-to-leading order (and chiral perturbation theory at leading order), within SM3 one has trouble giving the correct $\mathrm{Re}\,(\varepsilon'/\varepsilon)$ value. However, for SM4, solutions exist for $R_6 \sim 2.2$ and $R_8 = 0.8$–1.1, for $r_{ds} \sim (3.5–5) \times 10^{-4}$ and $\phi_{ds} \sim -(45^\circ–60)^\circ$, as illustrated by the elliptic rings on upper right part of Fig. 2. We will take

$$r_{ds} \sim 5 \times 10^{-4}, \quad \phi_{ds} \sim -60^\circ \text{ or } +35^\circ, \tag{15}$$

as our two nominal cases that satisfy all kaon constraints. The corresponding values for R_6 and R_8 can be roughly read off from Fig. 2. We stress again that these values should be taken as exemplary.

To illustrate in a different way, we plot ε_K, $\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})$ and $\mathrm{Re}\,(\varepsilon'/\varepsilon)$ vs ϕ_{ds} in Figs. 3(a), (b) and (c), respectively, for $r_{ds} = 4$ and 6×10^{-4}. The current 1σ experimental range is also illustrated. In Fig. 3(c), we have illustrated with $R_6 = 1.1$, $R_8 = 1.2$ [15] and $R_6 = 2.2$, $R_8 = 1.1$ [16]. For the former (latter) case, the variation is enhanced as R_6 (R_8) drops.

It is interesting to see what are the implications for the CPV decay $K_L \to \pi^0\nu\bar{\nu}$. The formula for $\mathcal{B}(K_L \to \pi^0\nu\bar{\nu})$ is analogous to Eq. (11), except [15] the change of κ_+ to $\kappa_L = (2.12 \pm 0.03) \times 10^{-10} \times (|V_{us}|/0.224)^8$, and taking only the imaginary part for the various CKM products. Since $\phi_{ds} \sim -60^\circ$ or $+35^\circ$ have large imaginary part, while $r_{ds} \equiv |V^*_{t'd}V_{t's}| \sim 5\times10^{-4}$ is stronger than the SM3

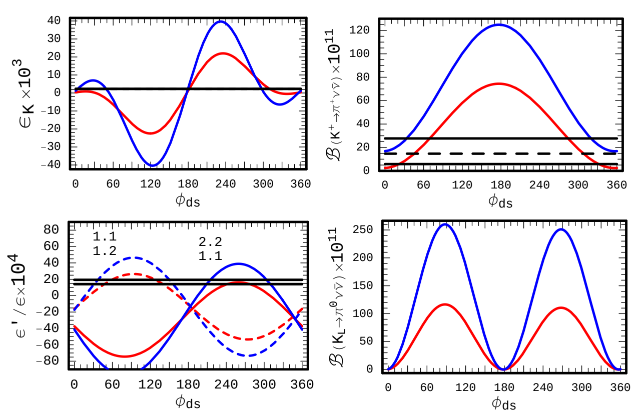

FIG. 3: (a) ε_K, (b) $\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})$, (c) $\mathrm{Re}\,(\varepsilon'/\varepsilon)$ and (d) $\mathcal{B}(K_L \to \pi^0\nu\bar{\nu})$ vs ϕ_{ds}, for $r_{ds} = 4$ and 6×10^{-4} and $m_{t'} = 300$ GeV. Larger r_{ds} gives stronger variation, and horizontal bands are current (1σ) experimental range [8] (the bound for (d) is outside the plot). For (c), solid (dashed) lines are for $R_6 = 2.2$, $R_8 = 1.1$ ($R_6 = 1.1$, $R_8 = 1.2$).

expectation of $\mathrm{Im}\, V_{td}^* V_{ts} \sim 10^{-4}$, we expect the CPV decay rate of $K_L \to \pi^0 \nu\bar{\nu}$ to be much enhanced.

We plot $\mathcal{B}(K_L \to \pi^0 \nu\bar{\nu})$ vs ϕ_{ds} in Fig. 3(d), for $r_{ds} = 4$ and 6×10^{-4}. Reading off from the figure, we see that the $K_L \to \pi^0 \nu\bar{\nu}$ *rate can reach above* 10^{-9}, almost two orders of magnitude above SM3 expectation of 0.3×10^{-10}. It is likely above 5×10^{-10}, and in general larger than $K^+ \to \pi^+ \nu\bar{\nu}$. Specifically, for our nominal value of $r_{ds} \sim 5 \times 10^{-4}$ and $\phi_{ds} \sim +35°$, $\mathcal{B}(K_L \to \pi^0 \nu\bar{\nu})$ and $\mathcal{B}(K^+ \to \pi^+ \nu\bar{\nu})$ are 6.5 and 2×10^{-10}, respectively, while for the $\phi_{ds} \sim -60°$ case, they are 12 and 3×10^{-10}, respectively. The latter case is closer to the Grossman-Nir bound [17], i.e. $\mathcal{B}(K_L \to \pi^0 \nu\bar{\nu})/\mathcal{B}(K^+ \to \pi^+ \nu\bar{\nu}) \sim \tau_{K_L}/\tau_{K^+} \sim 4.2$, because $V_{t'd} V_{t's}^*$ is more imaginary. Thus, both $K^+ \to \pi^+ \nu\bar{\nu}$ and $K_L \to \pi^0 \nu\bar{\nu}$ should be very interesting at the next round of experiments. We note that the ongoing E391A experiment could [18] attain single event sensitivity with the Grossman-Nir bound based on the current $\mathcal{B}(K^+ \to \pi^+ \nu\bar{\nu})$ measurement. However, for $r_{ds} \sim 3.5 \times 10^{-4}$ and $\phi_{ds} \sim -45°$, which is still a solution for $R_6 \sim 2.2$, one has $\mathcal{B}(K_L \to \pi^0 \nu\bar{\nu}) \sim 4 \times 10^{-10}$ with $\mathcal{B}(K^+ \to \pi^+ \nu\bar{\nu})$ at lower end of current range.

With $\phi_{sb} \sim 70°$ and $\phi_{ds} \sim -60°$ (and $+35°$) both sizable while the associated CKM product is larger than the corresponding SM3 top contribution, there is large impact on $b \to s$ and $s \to d$ transitions from Z penguin and box diagrams. It is therefore imperative to check that one does not run into difficulty with $b \to d$ transitions. Remarkably, we find that the impact on $b \to d$ is mild. From Eqs. (1), (8), (10) and (15), we infer

$$r_{db} \sim 1 \times 10^{-3}, \quad \phi_{db} \sim 10° \ (105°). \tag{16}$$

Since r_{db} is much smaller than $|V_{td}^* V_{tb}| \sim \lambda^3 \sim 0.01$ in SM3, the impact on $b \to d$ is expected to be milder, i.e. we are not far from the $V_{t'd} \to 0$ limit. We stress that this is *nontrivial* since there is a large effect in $b \to s$; it is a consequence of imposing $s \to d$ and $Z \to b\bar{b}$ constraints. We illustrate in Fig. 1(b) the unitarity quadrangle

$$V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* + V_{t'd} V_{t'b}^* = 0. \tag{17}$$

In contrast to Fig. 1(a), $(V_{td} V_{tb}^* + V_{t'd} V_{t'b}^*)_{\rm SM4}$ and $(V_{td} V_{tb}^*)_{\rm SM3}$ can hardly be distinguished.

The B_d^0-$\overline{B}_d^0$ mass difference and CP violation phase in mixing are respectively given by $\Delta m_{B_d} \equiv 2\,|M_{12}|$ and $\sin 2\Phi_{B_d} \equiv \mathrm{Im}\,(M_{12}/|M_{12}|)$, where

$$M_{12} = \kappa_{B_d} \Big[(\lambda_t^{db})^2 \eta_t S(x_t) + (\lambda_{t'}^{db})^2 \eta_{t'} S(x_{t'}) + 2 \lambda_t^{db} \lambda_{t'}^{db} \eta_{tt'} S(x_t, x_{t'}) \Big], \tag{18}$$

with $\kappa_{B_d} = \frac{G_F^2}{12\pi^2} m_W^2 m_{B_d} B_{B_d} f_{B_d}^2$. The functions $S(x)$ and $S(x,y)$ can be found in [19]. We take $\eta_t = 0.55$, $\eta_{t'} = 0.58$ and $\eta_{tt'} = 0.50$, and plot in Fig. 4(a) Δm_{B_d} vs ϕ_{db}, for $r_{db} = 8$ and 12×10^{-4} (corresponding to $r_{ds} = 4$ and 6×10^{-4}). We have taken the experimental value of

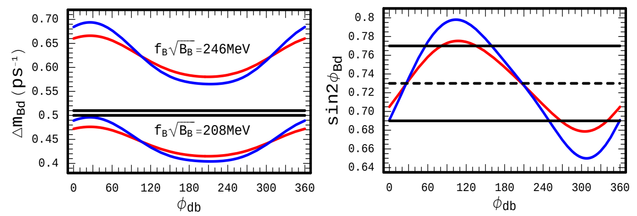

FIG. 4: (a) Δm_{B_d} and (b) $\sin 2\Phi_{B_d}$ vs ϕ_{db} for $r_{db} = 8$ and 12×10^{-4}, with $V_{t'd}^* V_{t'b} \equiv r_{db}\, e^{i\phi_{db}}$. Larger r_{db} gives stronger variation, and horizontal bands are the experimental range [8].

$\Delta m_{B_d} = (0.505 \pm 0.005)\ \mathrm{ps}^{-1}$ from PDG 2005 [8], and illustrated with the lower range of $f_{B_d}\sqrt{B_{B_d}} = (246 \pm 38)$ MeV [20]. We have scaled up the error for the latter by 1.4, since it comes from the new result on f_{B_d} with unquenched lattice QCD [21], but B_{B_d} is not yet updated. We see from Fig. 4(a) that Δm_{B_d} does not rule out the parameter space around Eq. (16) (equivalent to Eq. (15)). The overall dependence on r_{db} and ϕ_{db} is mild, and error on $f_{B_d}\sqrt{B_{B_d}}$ dominates. Seemingly, a lower value of $f_{B_d}\sqrt{B_{B_d}} \sim 215$ MeV is preferred. SM3 would give $\Delta m_{B_d} = 0.44 - 0.62\ \mathrm{ps}^{-1}$ for $f_{B_d}\sqrt{B_{B_d}} = 208$ MeV -246 MeV, so the problem is not with SM4.

We plot $\sin 2\Phi_{B_d}$ vs ϕ_{db} in Fig. 4(b), for $r_{db} = 8$ and 12×10^{-4}. One can see that $\sin 2\Phi_{B_d}$, which is not sensitive to hadronic parameters such as $f_{B_d}\sqrt{B_{B_d}}$, is well within experimental range of "$\sin 2\phi_1$"$= 0.73 \pm 0.04$ from PDG 2005 [8] for the $\phi_{db} \sim 10°$ case. However, for $\phi_{db} \sim 105°$ case, which is much more imaginary, $\sin 2\Phi_{B_d}$ is on the high side [22], and it seems that CPV in B physics prefers $R_6 \sim 2.2$ over $R_6 \sim 1$. As another check, we find the semileptonic asymmetry $A_{SL} = -0.7 \times 10^{-3}$ (-0.2×10^{-3}) for $\phi_{db} \sim 10°$ (105°), which is also well within range of $A_{SL}^{\rm exp} = (-1.1 \pm 7.9 \pm 7.0) \times 10^{-3}$ [23].

With Eqs. (1), (10) and (16), together with standard (SM3) values for V_{cb} and V_{ub}, we can get a glimpse of the typical 4×4 CKM matrix, which appears like

$$\begin{pmatrix} 0.9745 & 0.2225 & 0.0038\, e^{-i\,60°} & 0.0281\, e^{i\,61°} \\ -0.2241 & 0.9667 & 0.0415 & 0.1164\, e^{i\,66°} \\ 0.0073\, e^{-i\,25°} & -0.0555\, e^{-i\,25°} & 0.9746 & 0.2168\, e^{-i\,1°} \\ -0.0044\, e^{-i\,10°} & -0.1136\, e^{-i\,70°} & -0.2200 & 0.9688 \end{pmatrix} \tag{19}$$

for $\phi_{db} \sim 10°$ case (V_{cd} and V_{cs} pick up tiny imaginary parts, which are too small to show in angles). For the $\phi_{db} \sim 105°$ case, the appearance is almost the same, except $V_{td} \simeq 0.0082\, e^{-i\,17°}$ and $V_{ub'} \simeq 0.029\, e^{i\,74°}$. Note the "double Cabibbo" nature, i.e. the 12 and 34 diagonal 2×2 submatrices appear almost the same. This is a consequence of our choice of Eq. (10). To keep Eq. (1) intact, however, weakening s_{34} would result in even large $V_{t's}$, but it would still be close to imaginary. Since $V_{t^{(\prime)}d}^* V_{t^{(\prime)}s}$ are tiny compared to $V_{ud}^* V_{us} \simeq -V_{cd}^* V_{cs}$, the unitarity quadrangle for $s \to d$ cannot be plotted as in Fig. 1.

However, note that $V_{td}^* V_{ts}$ is almost real, and CPV in $s \to d$ comes mostly from t'.

The entries for $V_{ib'}$, $i = u,\, c,\, t$ are all sizable. $|V_{ub'}| \sim 0.03$ satisfies the unitarity constraint $|V_{ub'}| < 0.08$ [8] from the first row, but it is almost as large as V_{cb}. However, the long standing puzzle of unitarity of the first row could be taken as a hint for finite $|V_{ub'}| \sim 0.03$ [24].

The element $V_{cb'} \simeq -V_{t's}^*$ is even larger than V_{cb} and close to imaginary. Together with finite $V_{ub'}$, $V_{ub'}V_{cb'}^* \simeq 0.0033\, e^{-i\,5^\circ}$ $(0.0034\, e^{i\,9^\circ})$ is not negligible, and one may worry about D^0-$\bar{D}^0$ mixing. Fortunately the D decay rate is fully Cabibbo allowed. Using $f_D\sqrt{B_D} = 200$ MeV, we find $\Delta m_{D^0} \lesssim 0.05$ ps^{-1} for $m_{b'} \lesssim 280$ GeV, for both nominal cases of Eq. (16). Thus, the current bound of $\Delta m_{D^0} < 0.07$ ps^{-1} is satisfied, and the search for D^0 mixing is of great interest. This bound weakens by factor of 2 if one allows for strong phase between $D^0 \to K^-\pi^+$ and $K^+\pi^-$ [8].

If $m_{b'} < m_{t'}$, as slightly preferred by D^0-$\bar{D}^0$ mixing constraint, the direct search for b' just above 200 GeV at the Tevatron Run II could be rather interesting. Since $V_{cb'}$ is not suppressed, the b' quark would decay via charged current. Both b' and t', regardless of which one is lighter, with $m_{t'} \sim 300$ GeV and $|m_{t'} - m_{b'}| \lesssim 85$ GeV [8], can be easily discovered at the LHC.

The large and mainly imaginary element $V_{t's} \simeq -V_{cb'}^*$ in Eq. (19), being larger than V_{ts} and V_{cb}, may appear unnatural (likewise for $V_{ub'}$ vs V_{ub}). However, it is allowed, since the main frontier that we are just starting to explore is in fact $b \to s$ transitions. The current situation that $\mathcal{A}_{K^+\pi^-} \sim -0.12$ while $\mathcal{A}_{K^+\pi^0} \gtrsim 0$ in $B \to K\pi$ decays may actually be hinting at the need for such large $b \to s$ CPV effects. The litmus test would be finding Δm_{B_s} not far above current bound, *but with sizable* $\sin 2\Phi_{B_s} < 0$ [3], which may even emerge at Tevatron Run II. Our results studied here are for illustration purpose, but the main result, that $K_L \to \pi^0\nu\bar{\nu}$ may be rather enhanced, is a generic consequence of Eq. (1), which is a possible solution to the $B^+ \to K^+\pi^0$ DCPV puzzle.

In summary, the deviation of direct CPV measurements between neutral and charged B decays, $\mathcal{A}_{K^+\pi^0} - \mathcal{A}_{K^+\pi^-} \simeq 0.16$ while $\mathcal{A}_{K^+\pi^-} \simeq -0.12$, is a puzzle that could be hinting at New Physics. A plausible solution is the existence of a 4th generation with $m_{t'} \sim 300$ GeV and $V_{t's}^* V_{t'b} \sim 0.025\, e^{i\,70^\circ}$. If so, we find special solution space is carved out by stringent kaon constraints, and the 4×4 CKM matrix is almost fully determined. $K^+ \to \pi^+\nu\bar{\nu}$ may well be of order $(1-2) \times 10^{-10}$, while $K_L \to \pi^0\nu\bar{\nu} \sim (4-12) \times 10^{-10}$ is greatly enhanced by the large phase in $V_{t'd}^* V_{t's}$. With kaon constraints satisfied, B_d mixing and $\sin 2\Phi_{B_d}$ are consistent with experiment. Our results are generic. If the effect weakens in $b \to s$ transitions, the effect on $K \to \pi\nu\bar{\nu}$ would also weaken. But a large CPV effect in electroweak $b \to s$ penguins would translate into an enhanced $K_L \to \pi^0\nu\bar{\nu}$ (and $\sin 2\Phi_{B_s} < 0$).

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